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TITLE FOCK EXCHANGE IN MESON THEORIES OF NUCLEI

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FOCK EXCHANGE IN MESON THEORIES OF NUCLEI

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The Fock exchange term in meson field theories of nuclear systems is shown to arise from a two-loop ground-state self-energy diagram. Evaluation of this diagram gives the relativistic or semirelativistic analog of the Fock exchange enrgy; it differs from the nucleon-nucleon Fock energy in including retardation effects. In finite meson-field theories of nuclear systems, the variational nature of the meson-field analog of the Hartree-Fock energy functional can be further elucidated.

At present, in addition to the traditional view of the nucleus as inhabited by nucleons interacting via two-body potentials, there is the picture of the nucleus as consisting of nucleons and virtual mesons or meson field with interaction of the Yukawa type in which the nucleons emit and absorb virtual mesons. In this picture the Hamiltonian for nucleons interacting with a scalar meson field ϕ is of the form

$$H = T_{\mathrm{F}} + \int \omega(k) a^{\dagger}(\mathbf{k}) a(\mathbf{k})\, d\mathbf{k} - g \int \Psi^{\dagger}(\mathbf{r}) J \Psi(\mathbf{r}) \phi(\mathbf{r})\, d\mathbf{r} \quad , \tag{1}$$

where the scalar field $\phi(\mathbf{r})$ and the annihilation operator $a(\mathbf{k})$ are related in the standard way. The kinetic energy T_{F} of the nucleon field is

$$T_{\mathrm{F}} = \int \Psi^{\dagger}(\mathbf{r}) t_{\mathrm{F}} \Psi(\mathbf{r})\, d\mathbf{r} \quad . \tag{2}$$

The notation here is quite condensed or abbreviated in order to make the ideas clear; for example, the reader must supply his interpretation of t_{F} and J.

In treating nuclear systems that are described by the Hamiltonian of Eq. (1), the mean-field approximation to the ground-state of the system is generally used. The mean-field approximation to the ground-state energy of a system of A nucleons is

$$E_{\mathrm{MF}} = \sum_{i\ \mathrm{occ}} \langle t_{\mathrm{F}} \rangle_i - \frac{g^2}{8\pi} \int \langle \Psi^{\dagger} J \Psi \rangle_{\mathbf{r}} \frac{e^{-m|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \langle \Psi^{\dagger} J \Psi \rangle_{\mathbf{r}'}\, d\mathbf{r} d\mathbf{r}' \quad , \tag{3}$$

where

$$\langle \Psi^{\dagger} J \Psi \rangle_{\mathbf{r}} = \sum_{i\ \mathrm{occ}} f_i^{\dagger}(\mathbf{r}) J f_i(\mathbf{r}) \quad , \tag{4}$$

and the sums are over the occupied nucleon states. This expression for E_{MF} is easily recognized as just the Hartree energy of the system of nucleons interacting via the two-body potential

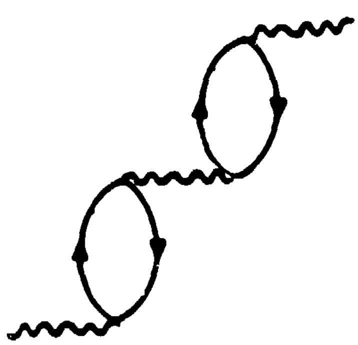
$$V(r) = -\frac{g^2}{4\pi} \frac{e^{-mr}}{r} \quad . \tag{5}$$

However, in the traditional picture, there would also be a Fock exchange term in the energy of the system; this term would be

$$+\frac{g^2}{8\pi}\sum_{ij\ \mathrm{occ}}\int f_j^{\dagger}(\mathbf{r})Jf_i(\mathbf{r})\frac{e^{-m|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}f_i^{\dagger}(\mathbf{r}')Jf_j(\mathbf{r}')\,d\mathbf{r}d\mathbf{r}' \quad . \tag{6}$$

The absence of a term corresponding to the Fock exchange term is clearly a problem of the mean-field approximation, although it seems not to have hindered the widespread application of the mean-field approximation to nuclear systems. Since the Fock exchange energy has a density dependence that differs from that of the Hartree potential energy, the presence or absence of a Fock exchange term will affect the density dependence of the energy of a nuclear system. It also affects the particular values of the coupling constants that give agreement of the theoretical and experimental values of binding energies and equilibrium shapes. For these reasons it is important to understand the status of the Fock exchange energy in the meson field theory of nuclear systems.

In order to see where the Fock exchange term might be in the meson-field approach to nuclear systems, it turns out to be useful to consider Gerry Brown's favorite diagram:



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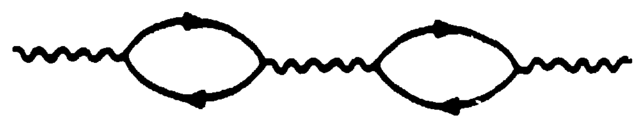
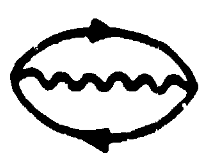


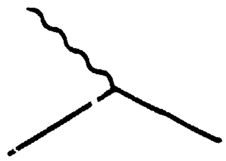
which is just the iteration of the simple particle-hole graph:

and if this is closed up, the result is the "hamburger" graph:



which is just a two-loop contribution to the ground-state energy of the system. In quantum electrodynamics it gives a contribution to the vacuum self-energy that never appears explicitly anywhere. Here in the case of nuclear systems it will be seen that the same diagram gives a term that is essentially the Fock exchange contribution to the energy.

The elementary meson-nucleon-nucleon vertex in the above diagrams is



with corresponding factor

$$V_{ij}(\mathbf{k}) = \int e^{-i\mathbf{k}\cdot\mathbf{r}} f_i^{\dagger}(\mathbf{r}) J f_j(\mathbf{r})\, d\mathbf{r} \quad , \tag{7}$$

and corresponding to the hamburger diagram is the expression

$$E_{\mathrm{ham}} = -g^2 \sum_{\substack{i \text{ unocc} \\ j \text{ occ}}} \int \frac{V_{ij}^*(\mathbf{k}) V_{ij}(\mathbf{k})}{16\pi^3 \omega(k)(\varepsilon_i + \omega(k) - \varepsilon_j)}\, d\mathbf{k} \quad ; \tag{8}$$

clearly this is just the sum of the self-energies of the occupied states,

$$E_{\mathrm{ham}} = \sum_{j \text{ occ}} S_j \tag{9}$$

$$S_j = \sum_{i \text{ unocc}} \int \frac{V_{ij}^*(\mathbf{k}) V_{ij}(\mathbf{k})}{16\pi^3 \omega(k)(\varepsilon_i + \omega(k) - \varepsilon_j)}\, d\mathbf{k} \quad . \tag{10}$$

However, this contribution to the energy must be renormalized to take into account the self energies the nucleons in occupied states would have if the other nucleons were not present. Clearly

$$S_j^{\mathrm{free}} = \sum_{i \text{ all}} \int \frac{V_{ij}^*(\mathbf{k}) V_{ij}(\mathbf{k})}{16\pi^3 \omega(k)(\varepsilon_i + \omega(k) - \varepsilon_j)}\, d\mathbf{k} \quad , \tag{11}$$

and therefore

$$\begin{aligned} E_{\mathrm{ham}}^{\mathrm{renorm}} &= \sum_{j \text{ occ}} (S_j - S_j^{\mathrm{free}}) \\ &= g^2 \sum_{ij \text{ occ}} \int \frac{V_{ij}^*(\mathbf{k}) V_{ij}(\mathbf{k})}{16\pi^3 \omega(k)(\varepsilon_i + \omega(k) - \varepsilon_j)}\, d\mathbf{k} \\ &= \frac{g^2}{16\pi^3} \sum_{ij \text{ occ}} \int \frac{V_{ij}^*(\mathbf{k}) V_{ij}(\mathbf{k})}{\omega^2(k) - (\varepsilon_i - \varepsilon_j)^2}\, d\mathbf{k} \quad . \end{aligned} \tag{12}$$

This expression is the meson-theory analog of the Fock exchange term of Eq. (6). The term $(\varepsilon_i - \varepsilon_j)^2$ in the denominator of Eq. (12) is a retardation correction to the usual Fock exchange energy; if it is neglected then $E_{\mathrm{ham}}^{\mathrm{renorm}}$ is easily seen to reduce to exactly the Fock exchange energy of Eq. (6). The term $(\varepsilon_i - \varepsilon_j)^2$ in the denominator of Eq. (12) is also just like the extra correction that Gerry Brown derived[1] in his "favorite paper" in 1952.

These results are not new (see Refs. 2 and 3), but the idea seems not to have been applied, so this is a welcome opportunity to outline its basic features. The above discussion applies to meson field theories with either nonrelativistic nucleons or relativistic nucleons. In the case of nonrelativistic nucleons in a finite meson field theory (Ref.32), it is possible to pursue the ramifications of the Fock exchange further and ask which energy functional is actually variational, the Hartree functional or the meson-theoretic Hartree-Fock functional. The details of the exploration of this question are too extensive to be exhibited in this symposium, but the new result is that by decomposing the field operators into internal and external parts, as in Ref. 4, it can be shown that the Hartree functional should be used to determine the single-particle orbitals, but the energy must then be corrected by adding the Fock exchange term computed with the Hartree orbitals.

To summarize, the Fock exchange term in meson field theories of nuclear systems comes from a two-loop ground-state self-energy diagram. Evaluation of this diagram gives the relativistic or semirelativistic analog of the Fock exchange enrgy; it differs from the nucleon-nucleon Fock energy in including retardation effects. In finite meson-field theories of nuclear systems, the diagrammatic understanding of the Fock exchange energy can be extended to provide a means of establishing the variational nature of the meson-field analog of the Hartree-Fock energy functional.

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